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Generalized Reciprocity

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The remarkable theorem of reciprocity as described by D. I. Hoult and R. E. Richards (*J. Magn. Reson.* 24, 71 (1976)) may be generalized to account for the near, intermediate, and radiation zone fields of a magnetic dipole. This form of reciprocity may be important when the wavelength of the NMR signal is not large compared to the distance scale of the system. In these situations the effects of interference may be significant. In addition, both the frequency dependence and distance dependence of the NMR signal amplitude are altered. In general, the distance dependence of the signal follows a weighted sum of $1/r^3$, $1/r^2$, and $1/r$ dependence. The frequency dependence of the signal amplitude is a function of ω, ω^2, and ω^3. Finally, the signal reflects the full vector field nature of the magnetic dipole. The mathematical expression of generalized reciprocity is completely equivalent to that of Hoult and Richards if the appropriate retarded potential form of the magnetic field is utilized.

Key Words: **NMR; signal; reciprocity.**

INTRODUCTION

The principle of reciprocity as described by Hoult and Richards (*1*) prescribes that the magnetic field produced at a point in space by a unit current in a radiofrequency (RF) coil is proportional to the electro-motive-force (EMF) induced in the coil by a magnetic dipole at the same point in space. If the field of the coil is expressed as a current normalized $\mathbf{B}_1$, the principle of reciprocity is given by

$$\mathscr{E} = -\frac{\partial}{\partial t}(\mathbf{B}_1 \cdot \mathbf{m}), \quad [1]$$

where $\mathbf{m}$ is the magnetic dipole moment and ε is the EMF induced in the coil. In short, the principle of reciprocity elucidates the equivalence of the transmission and reception fields of an RF coil. This equation allows a straightforward method of calculating the EMF induced in an RF coil and is therefore a fundamental step in calculating the signal level in any NMR experiment.

The principle of reciprocity as given by Eq. [1] where the $\mathbf{B}_1$ field is given by the law of Biot and Savart applies in the near-field limit of electromagnetic fields. This limit is valid when the distance scale of the experiment is much less than the wavelength of the electromagnetic field so that $kr \ll 1$ where k is the wavenumber and r is the experimental distance scale. For most of NMR this is an excellent approximation. There are certain NMR experiments, however, in which the near-field approximation no longer provides a complete description of the system. The wavenumber, k, is given by

$$k = \omega\sqrt{\epsilon\mu}, \quad [2]$$

where ω is the resonance frequency, μ is the permeability, and ϵ is the permittivity which may be complex in a lossy dielectric. In SI units $\mu = \mu_r\mu_0$ and $\epsilon = \epsilon_r\epsilon_0$. In an NMR sample, the permeability does not differ appreciably from the value for the vacuum unless it contains ferromagnetic or rare earth material. The permittivity, however, may be very different from that of the vacuum in many samples. In water, for example, the dielectric constant is roughly 80 over a wide range of frequencies. At 600 MHz in a pure water sample, therefore, the value of k is roughly 100 m^{-1}. In order to be in the near-field limit at this field the distance scale of the experiment must be much less than 1 cm. In biological specimens at frequencies of tens to hundreds of megahertz the dielectric constant is also on the order of 80 (*2–4*). At a magnetic field of 4 T in a biological sample, therefore, the value of k may be roughly 30 m^{-1}. In order to be in the near-field limit for such an experiment the distance scale of the experiment must be much less than 3 cm. Neither of these cases will be entirely described by a near field principle of reciprocity.

In this work we formulate a theory of generalized reciprocity (*5*). This theory of generalized reciprocity defines the EMF induced in an RF coil and is valid in the near, intermediate, and radiation zone fields of the magnetic dipole. Our result takes the form of

$$\mathscr{E} = -\frac{\partial}{\partial t}(\mathbf{m} \cdot \mathbf{B}'), \quad [3]$$

which is equivalent to the form originally presented by Hoult given in Eq. [1]. In the generalized theory of reciprocity,

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however, the form of the magnetic field from a current-carrying conductor is written as a generalized form of the law of Biot and Savart (*6, 7*)

$$\mathbf{B}' = \frac{\mu_0}{4\pi} \int e^{ikr}(1 - ikr)\, \frac{d\mathbf{l} \times \mathbf{r}}{|\mathbf{r}|^3}, \qquad [4]$$

where $\mathbf{B}'$ is the current-normalized magnetic field at the dipole which, without loss of generality, is placed at the origin. This form of the magnetic field reduces to the law of Biot and Savart in the limit where $kr \ll 1$. Our results predict that the signal intensity at high field can be higher than that predicted by the near field approximation and that the effects of interference may be observed.

THEORY

Near Field Reciprocity

Consider a loop of wire and an isolated magnetic dipole. Faraday's Law gives the EMF, $\mathscr{E}$, induced in the loop by the time varying magnetic field, $\mathbf{B}$, produced by the point magnetic dipole:

$$\mathscr{E} = -\frac{\partial}{\partial t} \int \mathbf{B} \cdot \hat{\mathbf{n}}\, dS. \qquad [5]$$

In this equation $\hat{\mathbf{n}}$ is the unit vector normal to the surface bounded by the loop of wire. Using the relationship between the magnetic field and the vector potential, $\mathbf{B} = \nabla \times \mathbf{A}$, where $\mathbf{A}$ is the vector potential, yields

$$\mathscr{E} = -\frac{\partial}{\partial t} \int (\nabla \times \mathbf{A}) \cdot \hat{\mathbf{n}}\, dS. \qquad [6]$$

Applying Stoke's theorem to convert the surface integral to a line integral yields an alternate form of Faraday's Law:

$$\mathscr{E} = -\frac{\partial}{\partial t} \int \mathbf{A} \cdot d\mathbf{l}. \qquad [7]$$

The vector potential of the magnetic point dipole, $\mathbf{m}$, at the origin is given by

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \mathbf{r}}{|\mathbf{r}|^3}, \qquad [8]$$

where $\mathbf{r}$ is the vector from the dipole to the position at which the vector potential is evaluated, and the magnetic dipole has a harmonic time dependence. It should be noted that this form of the vector potential includes only the near field contributions. Since only the vector potential at the loop of wire is of importance, $\mathbf{r}$ may be taken as the vector from the magnetic dipole to positions on the loop of wire. Substituting the form of the vector potential in Eq. [8] into Eq. [7] yields

$$\mathscr{E} = -\frac{\mu_0}{4\pi} \frac{\partial}{\partial t} \int \frac{\mathbf{m} \times \mathbf{r}}{|\mathbf{r}|^3} \cdot d\mathbf{l}. \qquad [9]$$

In order to satisfy the requirements of gauge invariance the loop over which the EMF is evaluated must be closed. Using the fact that $\mathbf{m} \times \mathbf{r} \cdot d\mathbf{l} = -\mathbf{m} \cdot d\mathbf{l} \times \mathbf{r}$, Eq. [9] may be expressed as

$$\mathscr{E} = \frac{\mu_0}{4\pi} \frac{\partial}{\partial t} \int \mathbf{m} \cdot \frac{d\mathbf{l} \times \mathbf{r}}{|\mathbf{r}|^3}. \qquad [10]$$

The magnetic dipole is a point dipole and, therefore, $\mathbf{m}$ may be taken out of the integral without loss of generality. This yields

$$\mathscr{E} = \frac{\mu_0}{4\pi} \frac{\partial}{\partial t} \mathbf{m} \cdot \int \frac{d\mathbf{l} \times \mathbf{r}}{|\mathbf{r}|^3}. \qquad [11]$$

The law of Biot and Savart indicates that the magnetic field, $\mathbf{B}$, at the origin produced by a uniform current, I, in the loop is given by

$$\mathbf{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\mathbf{l} \times \mathbf{r}}{|\mathbf{r}|^3}, \qquad [12]$$

where $\mathbf{r}$ is the vector from the conductor to the point where the field is evaluated. In this case, the field will be evaluated at the position of the magnetic dipole. Note that the direction of the vector $\mathbf{r}$ in Eq. [12] is reversed from the direction of $\mathbf{r}$ specified in Eqs. [8]–[11]. The current normalized magnetic field produced by current in the loop of wire at the position of the magnetic dipole is

$$\mathbf{B}' = \frac{\mu_0}{4\pi} \int \frac{d\mathbf{l} \times \mathbf{r}}{|\mathbf{r}|^3}. \qquad [13]$$

Reversing the orientation of $\mathbf{r}$ and substituting this expression into Eq. [11] yields

$$\mathscr{E} = -\frac{\partial}{\partial t} (\mathbf{m} \cdot \mathbf{B}'), \qquad [14]$$

which is equivalent to the near field form of the principle of reciprocity described by Hoult and Richards (*1*). A similar ''by Goldman et al.'' derivation appears elsewhere (*8*). In order to determine the total EMF induced in the loop from a macroscopic sample composed of magnetic dipole elements, Eq. [14] must be integrated over all space. This is expressed as

$$\mathcal{E} = -\frac{\partial}{\partial t}\int \mathbf{M}\cdot\mathbf{B}' d^3r, \qquad [15]$$

where **M** is the magnetization and the normalized magnetic field of the conductor must be evaluated at the position of every magnetic dipole in the macroscopic sample.

Generalized Reciprocity

Consider the fact that the transmission of radiofrequency fields from the coil to the sample and from the sample to the coil takes a finite period of time. Between different parts of the coil and sample, therefore, the signal may appear with different phase. This is true whenever the coil dimensions, sample sizes, and distances involved are on the order of the wavelength of the radiofrequency field or greater. In this case, the retarded potential formalism must be invoked to fully analyze any interference effects which may appear.

Assuming a point source, the full multipole expansion of the vector potential may be written in terms of spherical Bessel functions and spherical harmonics for all points exterior to the source. The magnetic component of the dipole order term may be extracted from the full multipole expansion of the vector potential for the magnetic dipole at the origin and is given by

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi}(\mathbf{r}\times\mathbf{m})\frac{e^{ikr}}{r^3}(ikr - 1), \qquad [16]$$

where **r** is the vector from the dipole to points on the coil, k is the wavenumber, and the magnetic dipole, **m**, has a harmonic time dependence (*9*). Substituting this form for the vector potential into the form of Faraday's Law given by Eq. [7] yields

$$\mathcal{E} = \frac{\mu_0}{4\pi}\frac{\partial}{\partial t}\int (\mathbf{r}\times\mathbf{m})\frac{e^{ikr}}{r^3}(1 - ikr)\cdot d\mathbf{l}. \qquad [17]$$

As before, in order to satisfy gauge invariance, the loop over which the EMF is evaluated must be closed. Simplifying yields

$$\mathcal{E} = \frac{\mu_0}{4\pi}\frac{\partial}{\partial t}\mathbf{m}\cdot\int e^{ikr}(1 - ikr)\frac{d\mathbf{l}\times\mathbf{r}}{|\mathbf{r}|^3}. \qquad [18]$$

Note that in the near field limit, $kr \ll 1$, Eq. [18] reduces to the form of the EMF given by Eq. [11]. The form of the magnetic field from a current source when considering the retarded potentials is given by Eq. [4]. Note again that the direction of the vector **r** in Eq. [4] is reversed from the direction of **r** specified in Eqs. [16]–[18]. Therefore, when considering the appropriate form of the magnetic field from a point magnetic dipole source the full field form of the principle of reciprocity may be expressed as

$$\mathcal{E} = -\frac{\partial}{\partial t}(\mathbf{m}\cdot\mathbf{B}'), \qquad [19]$$

where **B′** is the current normalized magnetic field produced by the loop of wire. In order to determine the total EMF induced in the loop from a macroscopic sample composed of magnetic dipole elements Eq. [19] must be integrated over all space. This is expressed as

$$\mathcal{E} = -\frac{\partial}{\partial t}\int \mathbf{M}\cdot\mathbf{B}' d^3r, \qquad [20]$$

where **M** is the magnetization and the normalized magnetic field of the conductor must be evaluated at the position of every magnetic dipole in the macroscopic sample. Therefore, if the correct form of the magnetic field of the coil is considered, the basic form of the principle of reciprocity as expressed by Hoult and Richards (*1*) still holds.

RESULTS

The conventional form of the law of Biot and Savart applies only in the zero frequency limit. This is a good approximation in the near field limit, where $kr \ll 1$. However, when the wavelength is not large compared to the distance scale of the problem the generalized form of the law of Biot and Savart (*6, 7*) given in Eq. [4] must be used. Note that by expanding the term $e^{ikr}(1 - ikr)$, which appears in both Eqs. [4] and [18], the near field approximation is correct to second order in the product of k and r. Therefore, even in cases where the product of k and r approaches unity, both the near field form of reciprocity and the conventional form of the law of Biot and Savart are remarkably good approximations (*6*).

To demonstrate the consequences of generalized reciprocity on the NMR signal, consider a circular loop of wire and a point magnetic dipole with simple harmonic time dependence of frequency ω nearby and on its symmetry axis as shown in Fig. 1. In the model system that has been proposed, Eq. [11] may be evaluated for the near field result and the EMF induced in the coil is given by

$$\mathcal{E} = \frac{\mu_0}{2}\frac{R^2 m_z i\omega}{r^3}e^{-i\omega t}, \qquad [21]$$

where R is the radius of the circular coil and $r = (R^2 + d^2)^{1/2}$ where d is the distance of the magnetic dipole from the center of the detection coil. In this case, the EMF is 90° out-of-phase with the magnetization, the distance dependence is $1/r^3$, and the induced EMF is proportional to ω.

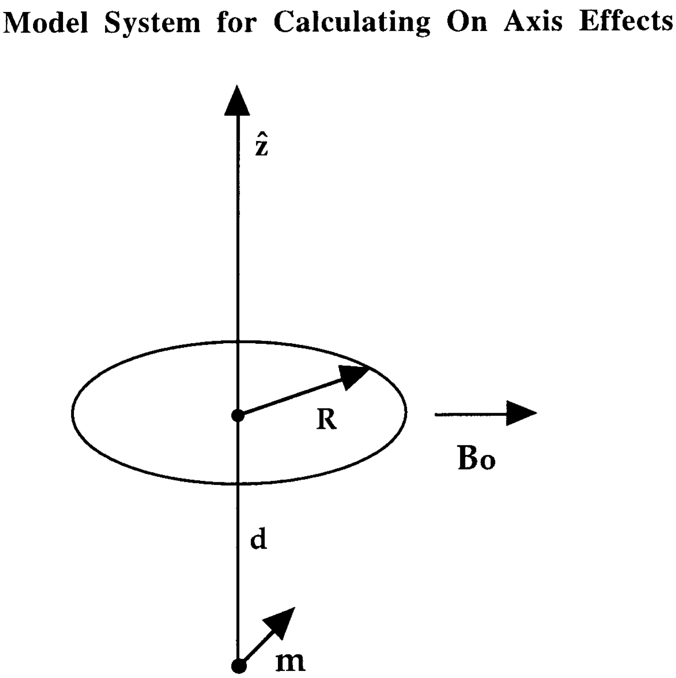

FIG. 1. A model system for calculating the on-axis effects of the full field form of reciprocity. The circular detection coil is of radius R and the magnetic dipole, $\mathbf{m}$, is a distance d from the center of the coil. The magnetic dipole is situated on the symmetry axis of the circular detection coil. All space is considered to be filled with a homogeneous and isotropic dielectric. The distance from any point on the detection coil to the position of the magnetic dipole is a constant in this system.

Note that only the component of the magnetic dipole parallel to the symmetry axis of the detection coil contributes to the EMF in this case.

If the effects of the retarded potentials are considered, then Eq. [18] must be evaluated and the EMF induced in the coil is

$$\mathscr{E} = \frac{\mu_0}{2}\frac{R^2 m_z i\omega}{r^3}(1 - ikr)e^{i(kr-\omega t)}. \quad [22]$$

This form of the EMF describes both the near and far field NMR effects and accounts for dielectric losses. In the near field, where $kr \ll 1$, the result given by Eq. [22] reduces to that given by [21]. Reciprocity as previously described is therefore a near field approximation of generalized reciprocity. In addition, generalized reciprocity explicitly accounts for dielectric losses in the form of a complex value of the wavenumber, k. If the wavenumber, k, is expressed as $\beta + i\alpha$ and substituted into Eq. [22] then the EMF is given by

$$\mathscr{E} = \frac{\mu_0}{2}\frac{R^2 m_z i\omega}{r^3}(1 - i\beta r + \alpha r)e^{i(\beta r-\omega t)}e^{-\alpha r}. \quad [23]$$

Examination of Eq. [23] reveals that a complex wavenumber both changes the phase of the EMF and introduces a distance-dependent decaying exponential.

For the purposes of simple comparison between generalized reciprocity and the near field approximation we address the case of no dielectric loss. In that case, the EMF induced in the coil is

$$\mathscr{E} = \frac{\mu_0}{2}\frac{R^2 m_z}{r^3}(i\omega + \sqrt{\epsilon\mu}\,\omega^2 r)e^{i(kr-\omega t)}, \quad [24]$$

where the permittivity, ϵ, is real. This form of the EMF may be easily compared to the near field result given in Eq. [21]. Comparison indicates that there is a change in the distance dependence of the signal, in the frequency dependence of the signal, and in the phase of the signal. In either case, only the component of the magnetic dipole parallel to the symmetry axis of the circular coil contributes to the EMF. Therefore, Eqs. [21] through [24] all hold for the case of a magnetic dipole rotating in the plane perpendicular to $\mathbf{B}_0$ as is conventionally assumed in magnetic resonance.

Figure 2 shows a comparison of the distance dependence of the signal specified by Eqs. [21] and [24] for a dielectric constant of 80 with no dielectric loss and a frequency of 178 MHz, the proton frequency at 4 T. The figure demon-

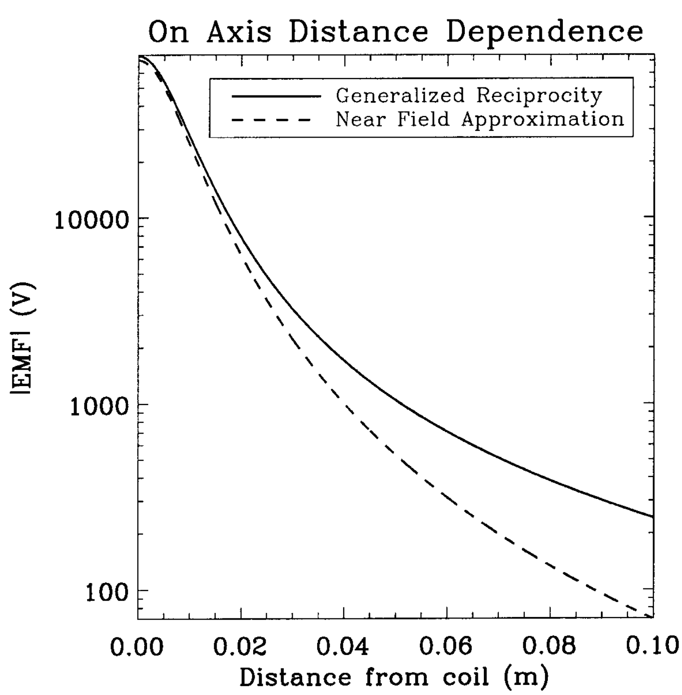

FIG. 2. The EMF induced by a dipole with magnetic moment 1 $A \cdot m^2$ as a function of the distance, d, between the dipole and the center of a 2-cm diameter circular coil as depicted in Fig. 1. For this example, the resonance frequency is chosen to be 178 MHz, the proton frequency at 4 T, with a dielectric constant of 80 and no dielectric loss. Far away from the detection coil the near field approximation predicts the signal falls off as $1/r^3$ while generalized reciprocity shows that the signal falls off as $1/r^2$.

strates that the signal falls off as $1/r^3$ in the near field approximation but as a combination of $1/r^2$ and $1/r^3$ in general. Note that when the magnetic dipole is on the axis of the circular coil no radiation zone signal, which would fall off as $1/r$, appears.

Figure 3 shows a comparison of the frequency dependence of the signal specified by Eqs. [21] and [24] for a magnetic dipole situated on the axis of a circular detection coil. The figure demonstrates that the signal increases proportionally to the frequency in the near field approximation but as a combination of ω and ω^2 in general. Note that the near field approximation is quite accurate until the frequency is approximately 60 MHz. At this frequency, the product of k and r is approximately equal to unity in this case. Note also when the magnetic dipole is on the axis of the circular coil no radiation zone signal, which is proportional to ω^3, appears.

For a magnetic dipole situated off the symmetry axis of the circular coil a solution is possible with a spherical harmonic expansion of the vector potential from the point dipole. The solution of Eqs. [11] and [18] for the case depicted in Fig. 4, where an off-axis point magnetic dipole with simple harmonic time dependence of frequency ω is considered, has been computed using numerical methods. Note again that only the component of the magnetic dipole parallel to the symmetry axis of the detection coil contributes to the EMF. Therefore, the solutions also hold for the case of a magnetic dipole rotating in the plane perpendicular to $\mathbf{B}_0$. Figure 5 shows, for a frequency of 178 MHz and a dielectric constant of 80 with no dielectric loss, the distance dependence of the signal. In the near field approximation the signal falls off as $1/r^3$ exterior to the detection coil. The full solution indicates that the signal falls off as $1/r$ far away from the coil.

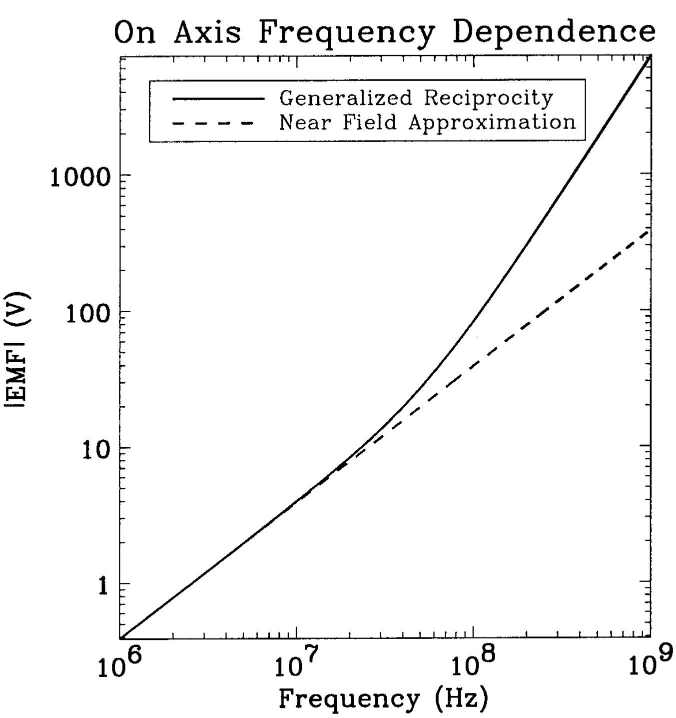

FIG. 3. The EMF induced by a dipole with magnetic moment 1 A · m^2 as a function of the resonance frequency situated at a distance of 10 cm from the center of a 2-cm diameter circular detection coil, as shown in Fig. 1. For this example, the dielectric constant is 80 and there is no dielectric loss. At high frequencies the near-field approximation predicts that the signal increases proportionally to ω, whereas generalized reciprocity shows that the signal increases proportionally to ω^2.

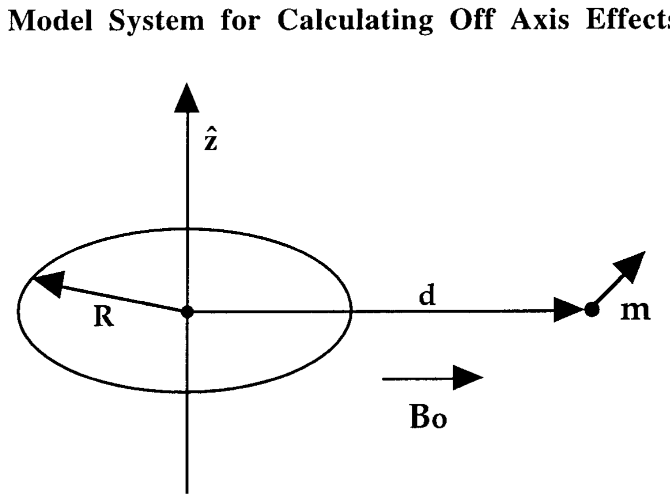

FIG. 4. A model system for calculating the off-axis effects of the full field form of reciprocity. The circular detection coil is of radius R and the magnetic dipole, **m**, is a distance d from the center of the coil. The magnetic dipole is situated in the plane of the circular detection coil. All space is considered to be filled with a homogeneous and isotropic dielectric. The distance from any point on the detection coil to the position of the dipole is not a constant in this system.

Interior to the detection coil the full field solution takes a much more complex character. As shown in Fig. 5, the signal drops below the level predicted by the near-field approximation. When the size of the detection coil is less than half of the wavelength of the NMR signal the effects of interference may be observed, and the signal may fall below the level predicted by the near-field approximation. In addition, multiple signal minima may occur interior to the coil which are separated from one another by a distance equal to half the wavelength of the NMR signal. Regardless, the signal levels at the minima never fall to zero in this case because the signals which add destructively to produce the minima have different amplitudes.

Figure 6 shows the frequency dependence of the signal for a point magnetic dipole off the axis of a circular detection coil as shown in Fig. 4. The signal in this case increases proportionally to ω in the near-field approximation but as a combination of ω, ω^2, and ω^3 in the general solution. Note that the near-field approximation is quite accurate until the frequency is approximately 40 MHz. The ω^3 frequency dependence of the signal in this case is due to the radiation zone field contribution to the signal.

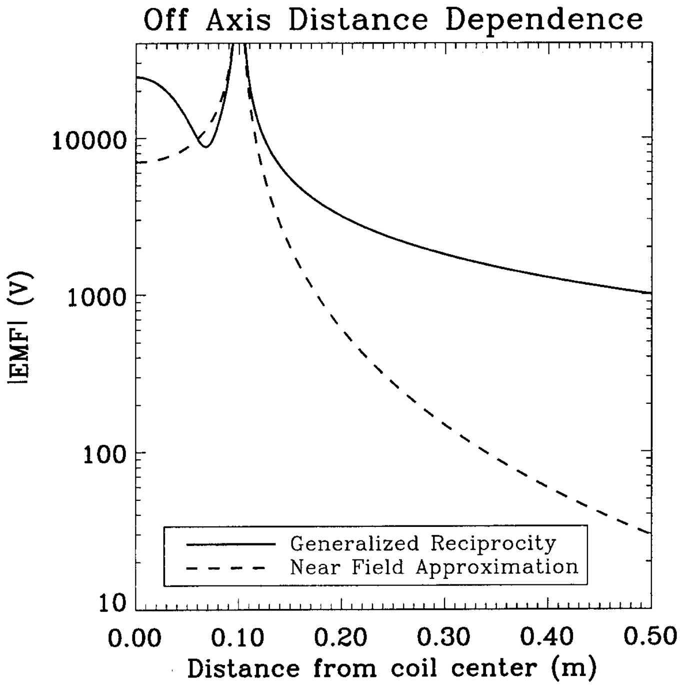

FIG. 5. The EMF induced by a dipole with magnetic moment 1 $A \cdot m^2$ as a function of the off-axis distance, d, between the dipole and the center of a circular coil as depicted in Fig. 4. For this example, the resonance frequency is chosen to be 178 MHz with a dielectric constant of 80 and no dielectric loss. The detection coil is taken to have a radius of 10 cm. Far away from the detection coil the near-field approximation predicts that the signal falls off as $1/r^3$ while generalized reciprocity shows that the signal falls off as $1/r$. Note that interior to the detection coil, generalized reciprocity demonstrates that the signal falls below the level predicted by the near-field approximation. This is due to an interference effect.

Exterior to the coil it is also possible to observe the effects of interference which arise from detection of the EMF with differing phase on different parts of the coil. In this case, the minima are separated from one another such that the difference in frequency corresponds to a wavelength which is equal to the diameter of the coil. The signal levels at the minima will never fall to zero in this case because the signals which add destructively have different amplitudes.

DISCUSSION

The near-field principle of reciprocity, as elucidated by Hoult and Richards (*1*), is an excellent approximation for most experimental cases considered in NMR. This is true for two reasons. First, most NMR experiments are performed at field strengths and distance scales in which the near-field limit holds. Second, even when the product of the wavenumber, k, and the distance scale, r, of the experiment approach unity the near-field approximation is quite accurate because the first nonzero correction terms to the near-field approximation are second order in the product of k and r.

In situations where the wavelength is not large compared to the distance scale, however, a generalized principle of reciprocity is required to describe any interference effects that may occur. In addition, the effects of the near, intermediate, and radiation zone fields of the magnetic dipole change the frequency dependence, distance dependence, phase, and vector nature of the signal. Finally, the near-field approximation of reciprocity makes no allowance for the effects of dielectric losses. Only a generalized theory of reciprocity will allow for a calculation of the signal that is valid in all experimental cases.

The calculation of the signal detected in high-field NMR experiments follows directly from a generalized principle of reciprocity. The signal, even for the simple case of an isolated magnetic dipole and a circular detection coil, exhibits many complex characteristics. As demonstrated in the two cases considered in this paper, the distance dependence of the signal is no longer the standard $1/r^3$ relationship. In general, the distance dependence of the signal may follow a $1/r^3$, $1/r^2$, or a $1/r$ dependence. This predicts an increased level of signal at higher field strengths. Also demonstrated is a change in the frequency dependence of the signal. In

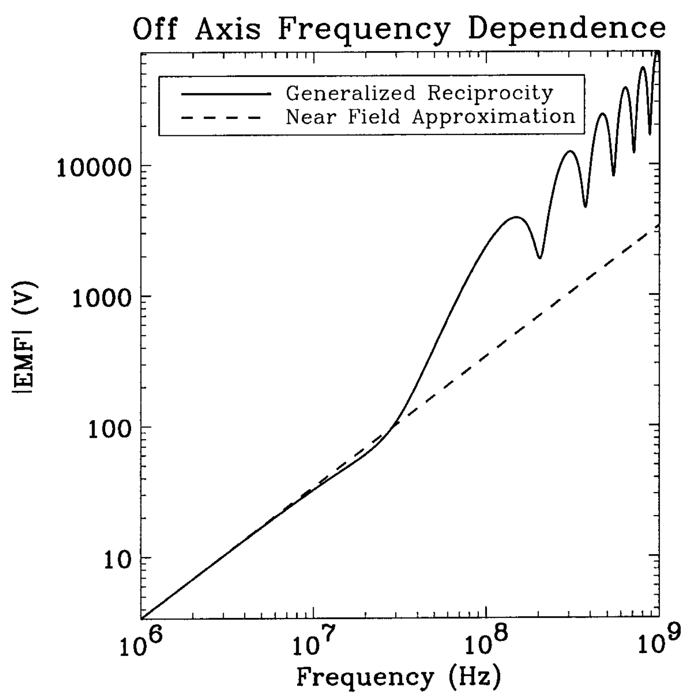

FIG. 6. The EMF induced by a dipole with magnetic moment 1 $A \cdot m^2$ as a function of the resonance frequency situated at a distance of 10 cm from the center of a circular detection coil, as shown in Fig. 4. The detection coil is taken to have a radius of 10 cm. For this example, the dielectric constant is 80 and there is no dielectric loss. At high frequencies, the near-field approximation predicts that the signal increases proportionally to ω, whereas generalized reciprocity shows that the signal increases proportionally to ω^3. In addition, interference effects, which decrease the signal, are observed.

the near-field case the signal is proportional to ω. In general, however, the signal may be proportional to ω, ω^2, or ω^3. This also implies that the signal increase at high field will be greater than that predicted by the near-field approximation.

The two cases considered in this paper also serve to demonstrate that the vector nature of the magnetic dipole must now be considered in a calculation of the signal. For example, in the case where the dipole is located on the symmetry axis of the detection coil as shown in Fig. 1, the signal follows an ω^2 frequency dependence and a $1/r^2$ distance dependence outside of the near field. This is because there is no on axis radiation from a magnetic dipole. When the dipole is situated off the axis of the detection coil as shown in Fig. 4, however, the effects of radiation from the magnetic dipole are now evident and the signal follows an ω^3 frequency dependence and a $1/r$ distance dependence outside of the near field. Thus, the vector nature of the field of a magnetic dipole must also be considered in a calculation of the signal in NMR.

Finally, the cases considered in this paper serve to demonstrate that signals may arrive at the detection coil with differing phases. Therefore, the interference of signals may occur. When a magnetic dipole is on the axis of a detection coil, as shown in Fig. 1, the signals from the dipole all arrive at the coil with the same phase as the distance from the dipole to all points on the coil is a constant. When the distance from the magnetic dipole to all points on the coil is not a constant, as in the case depicted in Fig. 4, the signal from the dipole arrives at the coil with a different phase at each point on the coil. Therefore, when the signals arrive at the coil with different phases the signals may interfere. This implies that the signal may fall below the level predicted by the near-field approximation.

The existence of finite conductivity produces dielectric loss. This decreases the signal levels attainable in any NMR experiment. The exact decrease in signal depends on both the conductivity and size of the sample. Dielectric loss also changes the phase of the signal. Therefore, the character of interference effects also depend in detail on the conductivity and size of the sample.

In summary, when the wavelength of the NMR signal approaches the distance scale of the experiment the predictions of the near-field approximation fail and a generalized theory of reciprocity is mandated. Generalized reciprocity reveals that the frequency and distance dependence of the signal differs from that previously assumed. Moreover, the signal reflects the full vector field nature of the magnetic dipole. Finally, when the wavelength of the NMR signal approaches the distance scale of the experiment, the effects of interference may be significant.

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